

The Golden Rectangle and Ratio

elongated



square

The ratio of the width of a golden rectangle to its height is called the *golden ratio*.

You can describe the shape of a rectangle using the ratio of its width to its height. The rectangle on the left has a ratio of about 5:1, and the square on the right has a ratio of 1:1. The shape in the middle is often considered to be more attractive and has been called the *golden rectangle*. Paintings, photos, books, and magazines are often made with proportions similar to the golden rectangle.

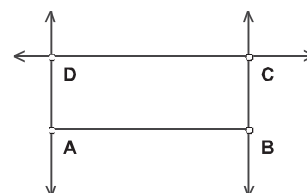
If you add a square to the long side of a golden rectangle, the result is still a golden rectangle, oriented vertically rather than horizontally. The new, larger rectangle still has its sides in the same ratio as the original. You'll use this property to construct a golden rectangle, determine its ratio of width to height, and explore its characteristics.



SKETCH AND INVESTIGATE

You'll begin by constructing an adjustable rectangle.

1. In a new sketch, use the **Segment** tool to draw segment AB .
2. Construct two lines perpendicular to segment AB , one through point A and the other through point B .
3. Construct point C on the line through B .
4. Construct a line parallel to segment AB through point C . Construct intersection D .
5. Hide the lines and construct segments to connect the four points.
6. Construct the quadrilateral interior.



7. Measure the lengths of segments AB and AD by selecting the segments and choosing **Measure | Length**. Calculate the ratio of the width to the height.

$$\begin{aligned} m\overline{AB} &= 4.48 \text{ cm} \\ m\overline{AD} &= 1.66 \text{ cm} \\ \frac{m\overline{AB}}{m\overline{AD}} &= 2.70 \end{aligned}$$

Q1 What point must you drag to adjust the rectangle's shape? Drag that point to make the shape more attractive to you. What's the ratio now?

With the **Arrow** tool, select the segment and both points. Then choose **Construct | Perpendicular Lines**.

Select the points in order and choose **Construct | Quadrilateral Interior**.

Choose **Number | Calculate** to show the Calculator. Click the measurements in the sketch to enter them into the Calculator.

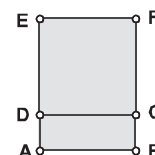
Use the Transform menu to mark a center and rotate a point.

Now you'll add a square above the rectangle.

8. Mark point D as the center of rotation, and rotate point C by 90° about D . Label the new point E .
9. Mark point E as the center of rotation, and rotate point D by 90° . Label the new point F .
10. You now have the four vertices of the added square. Construct the sides and interior of the square.

To measure the distance from A to E , select the two points and choose **Measure | Distance**.

11. The original rectangle and the new square together make a larger rectangle. Measure the longest side of this new rectangle. Then calculate the ratio of the longer and shorter sides of the large rectangle.



Q2 How does this ratio compare to the ratio from the original rectangle?

12. Adjust point C until the two ratios are as close to equal as you can make them.

Q3 What are the ratios now? This is the value of the golden ratio.

Q4 If you add another square on side AE to make a still larger rectangle, what do you think will be the ratio of the sides of this rectangle?

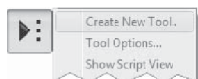
The golden ratio is often represented by the Greek letter ϕ (phi).

A GOLDEN SPIRAL

By constructing an arc inside square $CDEF$ and then adding more squares with arcs, you can construct a golden spiral.

13. Select in order points D , C , and E . Choose **Construct | Arc on Circle**.
14. Hide the labels of points C , D , F , and E .

Press and hold the **Custom** tool icon to show the Custom Tools menu.



15. Create a tool to make it easy to repeat the process: Select points C , D , F , and E , the segments connecting them, the square interior, and the arc. Then choose **Create New Tool** from the Custom Tools menu, and name the new tool **Square With Arc**.

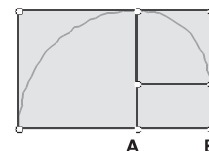


To use the new tool, choose it from the Custom Tools menu.

16. Use the new tool on points A and E to add a new square.

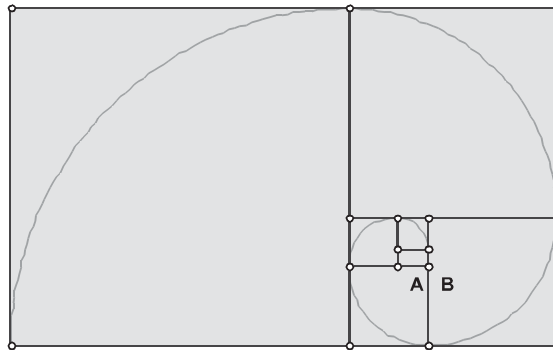
Q5 Measure the length and height of the new rectangle, and calculate their ratio. What result do you get?

To make more squares, you'll have to make the original shapes smaller.



Make sure each new arc connects to the previous arc.

17. Move *A* and *B* closer together, and use the new tool several times to add more squares to the existing rectangles.



- Q6** How many rectangles did you make?
- Q7** What do you think is the ratio of the sides of the largest rectangle you made? Measure the distances and calculate the ratio to confirm your conjecture.
- Q8** If your original rectangle was not a golden rectangle, what effect do you think there would be on the ratio of the sides of the largest rectangle? Drag point *C* to find out. What do you observe about the shape of the largest rectangle as you change the shape of the smallest one?

EXPLORE MORE

- Q9** Start a new spiral by using the **Square With Arc** tool twice at the same size before you start adding larger squares. As the rectangles get larger, what happens to the ratio of the sides?
- Q10** If the first square has sides of length 1, so does the second. How long are the sides of the third square? How long are the sides of the fourth square, and the fifth? Write down the sizes of the first 10 squares. Have you ever seen these numbers before?

